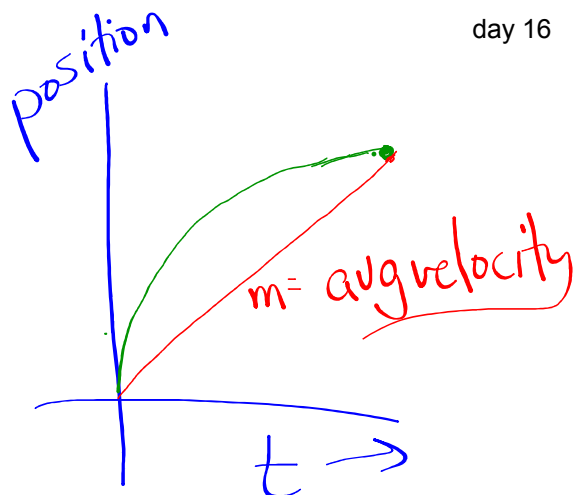


The derivative (3.1 - 3.2)

day 16

Average velocity

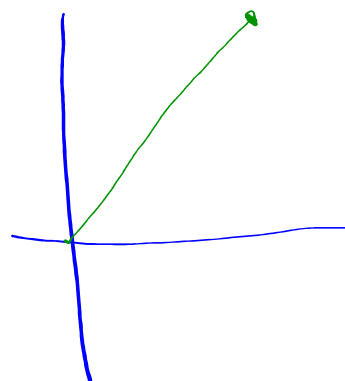
$$\frac{\text{chg in position}}{\text{chg in time}}$$



Instantaneous velocity

$$\lim_{t \rightarrow I} \frac{\text{chg in position}}{\text{chg in time}}$$

$$\lim_{t \rightarrow I} \frac{P(t) - P(I)}{t - I}$$



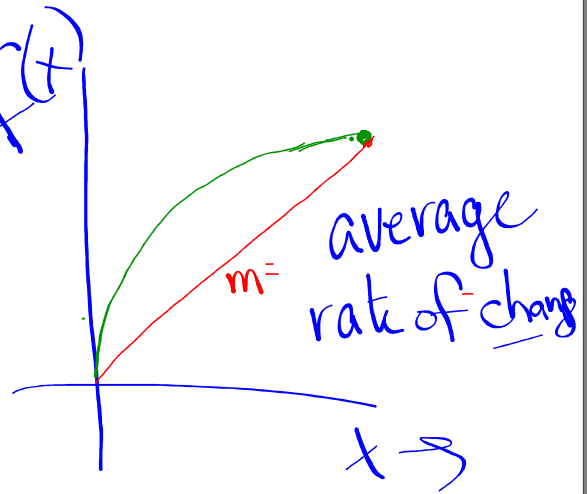
The derivative (3.1 - 3.2)

day 16

The average rate of change $f'(x)$
 of $f(x)$ between
 $x=a$ and $x=b$ is

a) the slope of the line
 joining $f(a)$ and $f(b)$

b)
$$\frac{f(b) - f(a)}{b - a}$$



The derivative

The derivative (3.1 - 3.2)

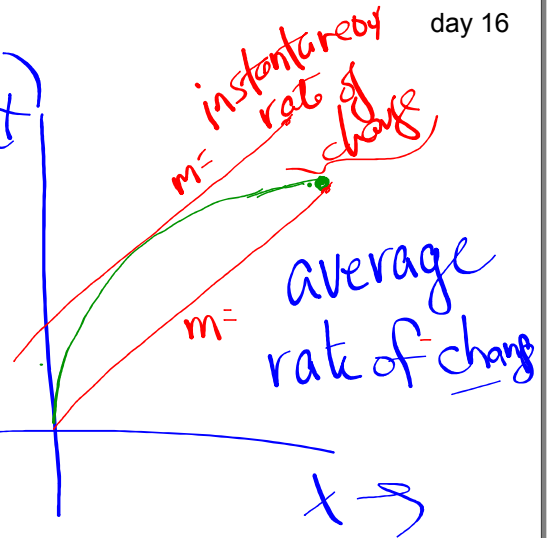
day 16

The instantaneous rate of change of $f(x)$ at

$$x = a$$

a) the slope of the TANGENT line AT $x = a$

b) $\lim_{b \rightarrow a} \frac{f(b) - f(a)}{b - a}$



The derivative

The derivative (3.1 - 3.2)

day 16

1) use def (1) ... to explain how slopes of
secant lines approach the slope
of the tangent line AT A POINT

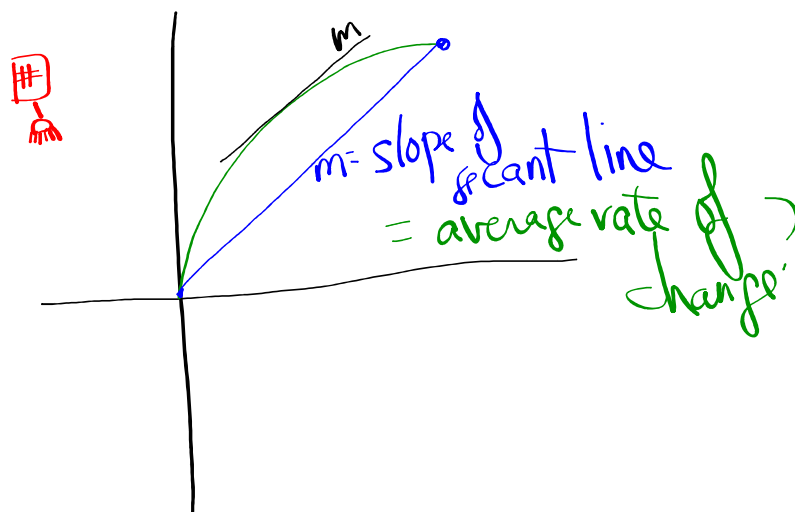
def 1) $m_{\text{TAN}} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

$x=a$

The derivative (3.1 - 3.2)

day 16

3.1/2 Explain WHY the slope of the secant line can be interpreted as average rate of change.



The derivative (3.1 - 3.2)

day 16

3.1/3 Explain WHY the slope of the tangent line can be interpreted as an instantaneous rate of change.

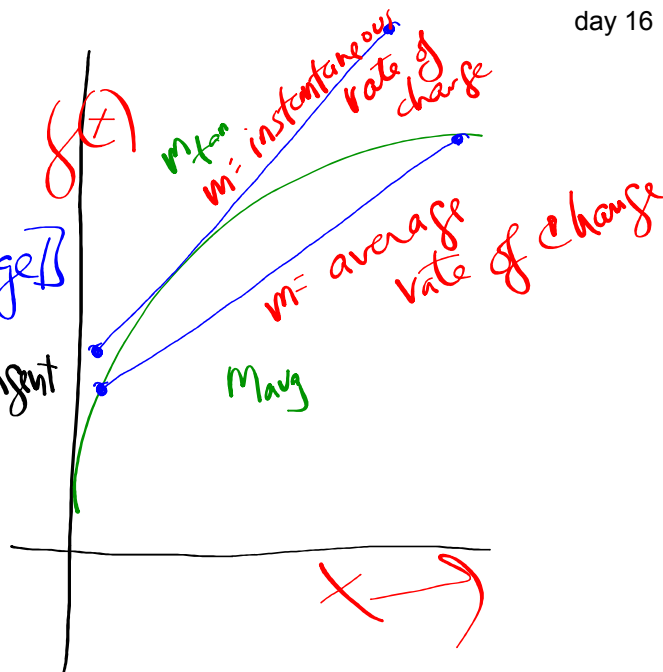
The derivative (3.1 - 3.2)

day 16

The derivative[[the instantaneous
rate of change]][[slope of the tangent
(line)]]

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

" f' " is called the derivative of $f(x)$



The derivative (3.1 - 3.2)

day 16

Equation of a line

"write the equation of the
line with slope = 2 through $P = (1, -7)$ "

$$y = mx + b$$

$$(y - (-7)) = 2(x - 1)$$

POINT-SLOPE form

$$y - y_1 = m(x - x_1) \text{ where } m \text{ is the slope}$$

(x_1, y_1) is ANY
point
on the line

The derivative (3.1 - 3.2)

day 16

$$3.1/9) f(x) = x^2 - 5, P(3, 4)$$

\uparrow
 $x=a$

$$f'(3) = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x^2 - 5) - (4)}{x - 3} = \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x \rightarrow 3} (x+3) = 6$$

$$f'(3) = 6, P(3, 4)$$

Pt-slope
formEqn
tan
line

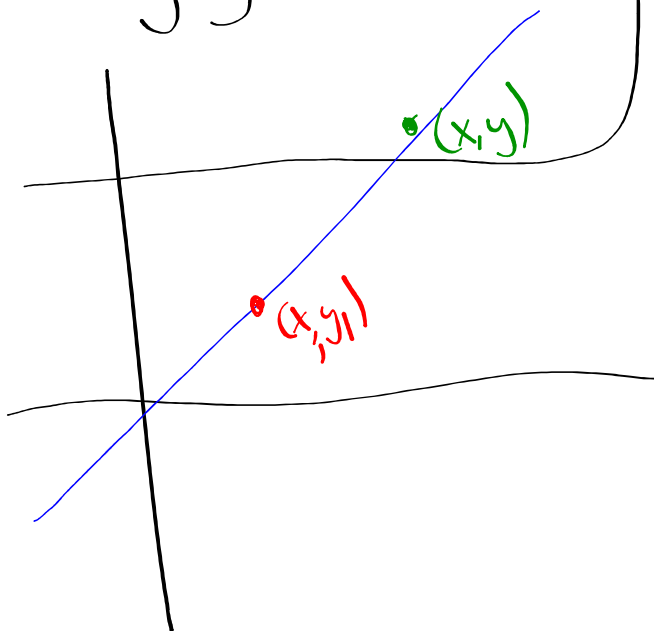
$$y - 4 = 6(x - 3)$$

The derivative (3.1 - 3.2)

day 16

pt slope

$$y - y_1 = m(x - x_1)$$



$$P_1 = (x, y), P_2 = (x_1, y_1)$$

$$m = \frac{y_1 - y}{x_1 - x} = \frac{y - y_1}{x - x_1}$$

$$\Downarrow$$

$$m(x - x_1) = y - y_1$$

The derivative (3.1 - 3.2)

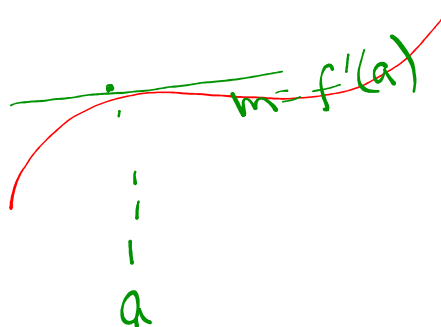
day 16

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

for the function $f'(x)$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{(x+h) - x} = h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$



The derivative (3.1 - 3.2)

day 16

Alternative notation

Gottfried Leibniz of Germany

2.6/71-73, 84, 86, 93
3.1/15, 17-20, 27-30, 37

$f'(x)$

$\frac{dy}{dx}$

"i.e. $\lim \frac{\Delta y}{\Delta x}$ "

if $y=f(x)$

\dot{f}

→ Newton
& Physics